Exercice 1 (DES - facteurs non linéaires) *Décomposer en éléments simples:*

1. $\dfrac{x}{(x+1)^2(x-2)}$
2. $\dfrac{x^2+3x+1}{x^2(x-1)}$
3. $\dfrac{x^2+1}{(x-1)^2(x+1)^2}$
4. $\dfrac{x^2+1}{(x-1)^2(x+2)}$
5. $\dfrac{x^2-1}{(x+2)^2(x+1)}$
6. $\dfrac{3x}{(x-1)^2(x+2)^2}$

Exercice 2 (DES réelle) *Faire la DES réelle des fractions rationnelles suivantes.*

1. $\dfrac{4\,x^2+6\,x+5}{(x^2+x+3)(x+1)}$
2. $\dfrac{2\,x^2-3\,x+3}{(x^2-x+2)(x-1)}$
3. $\dfrac{x^2+1}{(x^2+x-2)x}$
4. $\dfrac{x^2-15\,x+20}{(x^2-3\,x+4)(x+4)}$
5. $\dfrac{x-1}{(x^2+x+1)(x+2)}$
6. $\dfrac{x}{(x^2+x-6)(x+1)}$

Exercice 3 (DES - Calcul d'intégrales) *Calculer les intégrales suivantes:*

1. $\displaystyle\int_0^1 \frac{x}{(x+1)^2(x-2)}\,dx$
2. $\displaystyle\int_2^3 \frac{x^2+3x+1}{x^2(x-1)}\,dx$
3. $\displaystyle\int_2^4 \frac{x^2+1}{(x-1)^2(x+1)^2}\,dx$
4. $\displaystyle\int_{-1}^0 \frac{x^2+1}{(x-1)^2(x+2)}\,dx$
5. $\displaystyle\int_0^1 \frac{x^2-1}{(x+2)^2(x+1)}\,dx$
6. $\displaystyle\int_2^3 \frac{x}{(x-1)^2(x+2)^2}\,dx$

Commandes SAGE:

Exercice 1:

```
f(x)=(x^2+1)/((x-1)^2*(x+2))
show(f(x).partial_fraction())
```

$$\frac{5}{9\,(x+2)}+\frac{4}{9\,(x-1)}+\frac{2}{3\,(x-1)^2}$$

```
f(x)=(x^2-1)/((x+2)^2*(x+1))
show(f(x).partial_fraction())
```

$$\frac{1}{x+2}-\frac{3}{(x+2)^2}$$

```
f(x)=(3*x)/((x-1)^2*(x+2)^2)
show(f(x).partial_fraction())
```

$$-\frac{1}{9\,(x+2)}+\frac{1}{9\,(x-1)}-\frac{2}{3\,(x+2)^2}+\frac{1}{3\,(x-1)^2}$$

Exercice 2:

```
f(x)=(x^2-15*x+20)/((x^2-3*x+4)*(x+4))
show(f(x).partial_fraction())
```

$$-\frac{2\,(x-1)}{x^2-3\,x+4}+\frac{3}{x+4}$$

```
f(x)=(x-1)/((x^2+x+1)*(x+2))
show(f(x).partial_fraction())
```

$$\frac{x}{x^2+x+1}-\frac{1}{x+2}$$

```
f(x)=x/((x^2+x-6)*(x+1))
show(f(x).partial_fraction())
```

$$-\frac{3}{10\,(x+3)}+\frac{1}{6\,(x+1)}+\frac{2}{15\,(x-2)}$$

Exercice 3:

```
show(integral((x^2+1)/((x-1)^2*(x+2)),x,-1,0))
```

$$\frac{1}{9}\log(2)+\frac{1}{3}$$

```
show(integral((x^2-1)/((x+2)^2*(x+1)),x,0,1))
```

$$\log(3)-\log(2)-\frac{1}{2}$$

```
show(integral(x/((x-1)^2*(x+2)^2),x,2,3))
```

$$-\frac{1}{27}\log(5)+\frac{1}{9}\log(2)+\frac{2}{45}$$